1. Dimensions and Units

This presentation surveys the dimensions and units of primary importance in engineering. Such dimensions, after a system of units is established, become the variables of engineering mathematics.

1-1. Dimensions. Fundamental concepts are difficult to describe without establishing descriptive names to basic definitions. For example, what is mass, length, or time? If mass is defined as that which has inertia, what is inertia? In a dimensional system presented here these descriptive names are called dimensions. More precisely,

A dimension is a name given to a measurable quality or characteristic of an entity.

Dimensions are grouped into two categories, fundamental and secondary.

A fundamental dimension is accepted without definition.

To reduce the number of dimensions, certain descriptions can be expressed in terms of others.

A secondary dimension can be expressed by fundamental dimensions.

For example, length, area, and volume are describing certain characteristics of an object. But since an area is defined as measured as a length squared and a volume as a length cubed, in place of these dimensions all these descriptions can be stated in terms of some fundamental dimension, in this case, length. These are secondary dimensions derived from fundamental dimensions. For example,

$[A] = [L]^2$

The bracket is used to identify a dimension, and the above equation should be read, “The dimension of area is equivalent to the dimension of length squared”. Thus with these definitions, secondary or derived dimensions can be obtained from a small number of fundamental, basic, or primary dimensions.

For some concepts it is possible to derive secondary dimensions from the laws of nature. Newton's second law of motion requires that [F] = [M][a] = [M][L]/[t]^2. This definition presupposes three primary dimensions: mass, length, and time. Thus force is a secondary or derived dimension. Historically the dimension of force, [F], has been considered primary in the USGS or English dimensional system which complicates engineering calculations in the USCS by a factor of 32.2 (thoroughly discussed in Article 1-5).

1-2. Dimensional Systems. A dimensional system is an accepted standard of fundamental dimensions and derived dimensions with corresponding units and scales.

There are several dimensional systems in use throughout science and engineering. Fortunately for engineering students there is one dimensional system in use in most engineering applications. This is the SI system (abbreviated SI from French: *Système international d'unités*[1]) that was first adopted internationally and has now made its way into the United States. Unfortunately for engineering students the English dimensional system is still used occasionally as it is in common use throughout the non-scientific community in the United States. The English dimensional system is referred to as the USCS (United States Customary System) in some texts. Other dimensional systems include the mgs (millimeter-gram-second) system and cgs (centimeter-gram-second) system both used interchangeably as the metric system.

Fundamental dimensions in the SI system pertinent to thermodynamics include mass, length, time, and temperature. The dimension of force is derived through the application of Newton's law of motion. Fundamental dimensions in the USCS pertinent to thermodynamics include mass, length, time, temperature, and force. Thus a dimensional constant is required (Article 1-5).

All other dimensions are derived. Derived dimensions are dimensions that are defined by the application of fundamental dimensions. Examples are shown below.

derived dimension name	dimension	in terms of fundamental dimensions
pressure	[p]	[M] [t]$^{-1}$[L]$^{-1}$
volume	[V]	[L]^3
velocity	[*V*]	[L][t]$^{-1}$
acceleration	[a]	[L][t]$^{-2}$
energy, work, heat	[E]	[M][L]^2[t]$^{-2}$
power	[P]	[M][L]^2[t]$^{-1}$

[1] International Bureau of Weights and Measures, *The International System of Units (SI)* (8th ed.), ISBN 92-822-2213-6 (2006).

Only a few derived dimensions are listed above. There are many more which will be encountered in the study of thermodynamics and its application to engineering.

1-3. Units. While a dimension is a descriptive word picture, a unit is a definite standard or measure of a dimension:

A unit is an arbitrary amount of the quality to be measured with an assigned numerical value of unity.

For example the unit foot, yard, inch, rod, meter, and kilometer are all different units but with the common dimension of length. Units are arbitrarily defined by history and custom.

Historically the human body has been used to provide the basis for units of length. The foot was not standardized in ancient times and varied from city to city suggesting that the "foot" was actually a synonym for a "shoe". Archeologists believe that the Egyptians, Mesopotamians favored the cubit while the Romans and the Greeks favored the foot. Originally both the Greeks and the Romans subdivided the foot into 16 digits, but in later years, the Romans also subdivided the foot into 12 *unica* (from which both the English words "inch" and "ounce" are derived).

Of course, modern measures of units based upon repeatable and reliable standards have been in use since the Renaissance period (eighteenth century) and these standards have been constantly improved.

The French originated the meter in the 1790s as one/ten-millionth of the distance from the equator to the North Pole along a meridian through Paris. It is realistically represented by the distance between two marks on an iron bar kept in Paris at constant temperature. The International Bureau of Weights and Measures, created in 1875, upgraded the bar to one made of 90 percent platinum/10 percent iridium alloy. In 1960 the meter was redefined as 1,650,763.73 wavelengths of orange-red light, in a vacuum, produced by burning the element krypton (Kr-86). More recently (1984), the Geneva Conference on Weights and Measures has defined the meter as the distance light travels, in a vacuum, in 1/299,792,458 seconds with time measured by a cesium-133 atomic clock which emits pulses of radiation at very rapid, regular intervals. None of the definitions changed the length of the meter, but merely allowed this length to be duplicated more precisely.

Our English foot has not been so constant. The U. S. Congress legalized the use of the metric system in 1866 on the basis that one meter is exactly equal to 39.37 inches. In 1959 a number of English-speaking countries agreed that an inch is exactly equal to 2.54 centimeters so that the International foot is exactly equal to 0.3048 meters. The United States retained the old

1866 equivalency and called it the U. S. Survey foot so that 1 U. S. Survey foot equals 1.000002 International feet.

Other standards exist for units of the fundamental dimensions of mass, time, temperature, etc. and for the sake of brevity they are not repeated here. For these definitions refer to NIST Special Publication 330 (SP 330), *The International System of Units (SI)*.

Complicating matters is the use of confusing units in the USCS where the unit 'pound' is used interchangeably for both mass and force in colloquial English. In scientific and engineering applications this leads to what has been referred to as 'mass confusion'. In the scientific community the units of mass and force in the USCS are the pound-mass, abbreviated lb_m, and the pound-force, abbreviated lb_f.

In engineering calculations the unit of lb <u>must</u> always be written lb_m and lb_f as there is a significant difference between mass and force.

Dimension	SI dimensional system	SI unit	abbr.	USGS dimensional system	USGS unit	abbr.
Mass [F]	fundamental	kilogram	kg	fundamental	pound-mass	lb_m
Length [L]	fundamental	meter	m	fundamental	foot	ft
Time [t]	fundamental	second	s	fundamental	second	s
Force [F]	derived	Newton	N	fundamental	pound-force	lb_f
Temperature[2] [T]	fundamental	Kelvin	K	fundamental	Rankine	R

1-4. Scales. A scale is assigned to a dimension within a given dimensional system to indicate a magnitude of the dimension.

A scale is a rule or formula used to measure the magnitude of a dimension.

Most scales are linear. The magnitude difference between integers on the scale is the same. For instance, the distance or magnitude difference between 1 meter to 2 meters and 6 meters and 7 meters is the same. Some scales are not linear but logarithmic or otherwise. For instance on the Richter scale which is a measure earthquake strength, the distance or magnitude difference between 1 and 2 is considerably different than that between 8 and 9, the former being a mild or hardly perceptible shake whereas the latter is chaotic destruction. The decibel scale used to measure sound intensity is another example of a logarithmic scale.

[2] To further complicate matters for temperature, an accepted SI unit is Celsius, C, and an accepted USCS unit is Fahrenheit, F. These are not absolute units as K=C+273 and R=F+460. Absolute units should always be used to avoid calculation errors.

1-6. Dimensional homogeneity and Units. Dimensional homogeneity is demanded in all engineering calculations. This is often thought of as a scourge to students in engineering as the primary mission of students is to arrive at a correct numerical answer and then assign the appropriate unit(s) after the calculation is completed. This is incorrect. Dimensional homogeneity must be incorporated in all engineering calculations throughout.

Dimensional homogeneity is achieved through the use of simple unit conversions and dimensional constants. For instance, the calculation

$$10\frac{m}{s} \times \frac{km}{1000\,m} \times \frac{3600\,s}{hr} = 36\frac{km}{hr} \qquad [L][t]^{-1}$$

incorporates the use of a simple unit conversion. This unit conversion is simply

$$\frac{km}{1000\,m} = 1 \text{ and } \frac{3600\,s}{hr} = 1$$

Since both unit conversions equal 1 then they can be multiplied on either side of the equation without concern of equality (upsetting the equal sign). Other simple examples follow.

$$F = ma = 500\,kg \times 9.18\frac{m}{s^2} \times \frac{N{\cdot}s^2}{kg{\cdot}m} \times \frac{kN}{1000N} = 4.59\,kN \approx 4.6\,kN \qquad [M][L][t]^{-2}$$

$$p = \frac{4000N}{0.25m^2} \times \frac{m^2 Pa}{N} \times \frac{kPa}{1000\,Pa} = 16\,kPa \qquad [M]\,[t]^{-1}[L]^{-1}$$

$$K.E. = m\frac{V^2}{2} = 5\,kg\,\frac{40^2}{2}\frac{m^s}{s^2} \times \frac{N{\cdot}s^2}{kg{\cdot}m} \times \frac{J}{N{\cdot}m} \times \frac{kJ}{1000J} = 4\,kJ \qquad [M][L]^2[t]^{-2}$$

Note the last example often leads to error because students (and practicing engineers) that do not incorporate dimensional homogeneity throughout their calculations but simply ‘fake it’ by assigning units at the end of this calculation leave off the J to kJ conversion! This kinetic energy calculation is frequently encountered in First Law applications and lethargic students inevitably arrive at an incorrect answer.

1-5. Mass Confusion and the Dimensional Constant g_c. In USCS the use of a dimensional constant, called g_c, is often encountered in the quest for dimensional homogeneity. The dimensional constant g_c is not a unit conversion as force and mass in the USCS are both fundamental dimensions. The use of g_c is often confusing and sometimes left off of engineering calculations leading to a significant numerical error in the resulting calculations. It is, of course, absolutely essential. The dimensional constant g_c can be derived from the proportionality between force and mass through application of Newton’s law of motion.

$$F \sim mass \times acceleration$$

$$\frac{F}{F'} = \frac{ma}{m'a'}$$

$$F = \frac{1}{\frac{m'a'}{F'}} ma$$

$$\frac{F}{1\ lb_f} = \frac{ma}{32.174\ lb_m \frac{ft}{s^2}}$$

$$F = \frac{1}{32.174 \frac{lb_m\, ft}{lb_f\, s^2}} ma$$

$$F = \frac{1}{g_c} ma$$

$$g_c = 32.174\ \frac{lb_m\, ft}{lb_f\, s^2} \qquad [M][L]/[F][t]$$

Note that g_c is not the acceleration of gravity because it has different dimensions (although, unfortunately, the number 32.174 is equal to that of the standard acceleration of gravity in USCS units).

Example 1. Determine the force to accelerate 10 lb_m at the rate of 10 ft/s^2.

$$F = \frac{1}{g_c}\ 10\ lb_m\ 10 \frac{ft}{s^2} = \frac{100\ ft/s^2}{32.174\ \frac{lb_m\, ft}{lb_f\, s^2}} = 3.105\ lb_f$$

Example 2. Determine the force exerted by 10 lb_m in the earth's gravitational field at a location where $g = g_c$.

$$F = \frac{1}{g_c}\ 10\ lb_m\ 32.174 \frac{ft}{s^2} = \frac{321.74\ ft/s^2}{32.174\ \frac{lb_m\, ft}{lb_f\, s^2}} = 10\ lb_f$$

Example 3. Determine the kinetic energy in Btu of a 5 lb_m block moving at a velocity of 100 ft/s.

$$KE = \frac{1}{2} mV^2 = \frac{1}{2g_c}\ 5\ lb_m\ 10000 \frac{ft^2}{s^2} = 50000 \frac{lb_m ft^2}{s^2} \times \frac{1}{32.2} \frac{lb_f\, s^2}{lb_m ft} \times \frac{1\ Btu}{550\ ft\ lb_f} = 2.8\ Btu$$

Here g_c is numerically approximated at 32.2 which is a common approximation since engineering is an approximate science unlike physics or chemistry usually limited to a small number of significant figures (re: Article 1-7 below). Also, the use of g_c in the USCS is essential, for without it the calculation would be off by a factor of 32.2 (a common error amongst engineering students and entry level engineers)!

Note that the dimension of mass can be eliminated from a quantity by dividing by g_c, and the dimension of force can be eliminated from a quantity by multiplying by g_c.

1-6. Weight. Weight is a common measure familiar to anyone who has stepped on a bathroom scale. Weight is the measure of the force exerted on a body having mass by the acceleration of a gravitational field, usually the earth ($g_{standard}$ = 9.80665 m/s^2). The earth's acceleration of gravity is not constant but varies from location to location as the diameter and composition of the earth is not consistent throughout. It is usually approximated at 32.2 ft/s^2.

There is little confusion in the SI system because weight is force having units of Newtons, N or kN, whereas mass in the SI system has units of kilograms, kg. There is much confusion in the CSCS because of common use of the pound for both force and mass in colloquial English, with little or no differentiation between the actual units of lb_f and lb_m. In engineering and science applications the differentiation is demanded as illustrated in Examples 1 and 2 above. Common household items are often sold in pounds. For instance, a pound of butter purchased at the market means a pound-mass of butter, even though the label reads "lb" or "pounds". Compounding the measurement is the use of "ounces" which is used for both mass and volume in colloquial English! (An ounce and a fluid ounce are not the same thing!). When a person weighs himself or herself on a scale does that person wish to know his/her pound-mass or pound-force? Both may be of importance since the force exerted on a person's mass may be important to compare muscle strength (measured in pound-force) whereas total mass may be important to compare the actual magnitude of the dimension mass.

Further confusion arises when a person measures his or her weight on a bathroom scale (usually a spring scale). Here, force is actually measured (F=kx where k is the spring constant) and the gravitational field is inherent in the readout. When a person measures his or her weight on a beam balance (a doctor's scale) true mass is measured, independent of the gravitational field (why?). Again in the SI system this confusion is eliminated (unless the bathroom spring scale is incorrectly calibrated in kg!).

Some engineering disciplines, civil engineering for instance, used the unit of slug as a measure of mass. By definition, one pound-force (usually reported as pound) equals one slug multiplied by one ft/s^2. The use of g_c is eliminated. Note this is not the USCS but the Imperial System and the use of slug has pretty much fallen by the wayside.

1-7. Significant Figures. The significant figures (also called significant digits) of a number are those digits that carry meaning contributing to its precision. This includes all digits *except*:

- leading and trailing zeros where they serve merely as placeholders to indicate the scale of the number.
- spurious digits introduced, for example, by calculations carried out to greater accuracy than that of the original data, or measurements reported to a greater precision than the equipment supports.

The concept of significant digits is often used in connection with rounding. Rounding to *n* significant digits is a more general-purpose technique than rounding to *n* decimal places, since it handles numbers of different scales in a uniform way. For example, the population of a city might only be known to the nearest thousand and be stated as 52,000, while the population of a country might only be known to the nearest million and be stated as 52,000,000. The former might be in error by hundreds, and the latter might be in error by hundreds of thousands, but both have two significant digits (5 and 2). This reflects the fact that the significance of the error (its likely size relative to the size of the quantity being measured) is the same in both cases.[3]

The practice of engineering often involves data from gauges, readouts, and other output devices with varying degrees of accuracy. Oftentimes, three or four significant figures are all that are necessary because of these measurement devices. In some instances, for example pressure measurement with a Bourdon type gauge, two significant figures are all that one can expect from visual observation of the gauge faceplate.

Using more significant figures than implied by the data provides a false sense of accuracy.

Engineering students often report calculations to eight or nine significant figures because they have not developed a sense of the engineering profession and, of course, hand calculators provide this false accuracy. Engineering students also incorrectly think that it is better to report every digit rather than leave a few off the final answer with homework and quizzes. An answer reported with excess significant figures is incorrect.

The engineering profession is best served when data, calculations, predictions, and designs incorporate true accuracy with attention to tolerance intervals and significant figures.

[3] This illustrative example was paraphrased from Wikipedia http://en.wikipedia.org/wiki/Significant_figures on December 20, 2011. Similar texts are found in almost every introductory engineering text.