

5. Energy Analysis of Systems

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5.1. Time as the Only Independent Variable.

Consider an element of very small mass within a specified region of study. The element of mass m within the closed system has internal energy, and also it may have kinetic and potential energies. The total energy of the element, designated by the letter e , is

$$e = u + \frac{v^2}{2} + gZ \quad \left[\frac{\text{energy}}{\text{mass}} \right] \quad \text{Eq. a}$$

Each element within the area of study may have different energy values from its neighbors, especially when a process is underway. Therefore, at a specified time t , the energy of the entire system is indicated by a volume or space integral:

$$\begin{aligned} E|_t &= \int_{\text{volume at } t} e \rho dV \\ &= \int_{\text{volume at } t} e dm \quad [\text{energy}] \quad \text{b} \end{aligned}$$

The change in energy of the area under study in the time interval t_1 to t_2 is

$$E|_{t_1 \text{ to } t_2} = \int_{\text{volume at } t_2} e dm - \int_{\text{volume at } t_1} e dm \quad \text{c}$$

Notice that e may depend on both time and position, while E is a function of time alone. When e varies with position in a chaotic manner (the non-equilibrium case found during the usual dynamic process) evaluation of equations (a) and (b) may not be possible. But if the area under study is in equilibrium or near equilibrium at the beginning and end of the process, or if appropriate average values are considered for u , v , and Z , Equation (b) integrates to

$$E|_t = U + \frac{mV^2}{2} + mgZ$$

Similarly Equation (c) integrates to

$$E|_{t_1 \text{ to } t_2} = U_2 - U_1 + \frac{m(V_2^2 - V_1^2)}{2} + mg(Z_2 - Z_1)$$

In effect, this procedure of assuming the area under study is in equilibrium or near equilibrium at the beginning and end of the process reduces the independent variables to that of time alone.

The variables describing the closed system are selected so that time (not position) is the only independent variable.

5.2. The Energy Balance.

Moving from an area of study to a thermodynamic system (by definition of the system), the energy balance between system and surroundings demanded by the conservation of energy (First Law) for all processes is

$$-\Delta E_{surroundings} = \Delta E_{system} \quad f$$

Equation f can be changed into an equation involving only the system by employing the concepts of heat and work to replace $-\Delta E_{surroundings}$

$$\int_{t_1}^{t_2} \int_{surface} \frac{\partial^2 Q}{\partial t \partial A} dA dt - \int_{t_1}^{t_2} \int_{surface} \frac{\partial^2 W}{\partial t \partial A} dA dt = \int_{volume at t_2} edm - \int_{volume at t_1} edm \quad g$$

Note that both heat and work are surface phenomena with time as the independent variable. Note also the usual sign convention of (+)heat into the system and (+)work out of the system.

5.3. The First Law for the Closed System.

Equation g is an energy balance over the system alone. Surface integrations for heat and work and volume integrations for energy of the system are evaluated over the same time period t1 to t2. This is the general energy equation for analysis of the closed system.

In incremental form

$$\delta Q_{surface, t_1 to t_2} - \delta W_{surface, t_1 to t_2} = dE_{system, t_1 to t_2} \quad h$$

Where δ indicates a path integral (value is path dependent) and dE indicates a path independent the value.

Integrating Equation h over time:

$$\int_{t_1}^{t_2} \delta Q_{surface} - \int_{t_1}^{t_2} \delta W_{surface} = \int_{t_1}^{t_2} dE_{system} \quad i$$

Yields the First Law for the closed system (no mass transfer):

$$Q_1^2 - W_1^2 = \Delta E = U_2 - U_1 + \frac{m(v_2^2 - v_1^2)}{2} + mg(Z_2 - Z_1) \quad j$$

Where Q and W are dependent upon the path (heat and work are dependent upon the transfer process) and E is dependent only upon the initial and final states (as energy E is a function of system properties).

Equation j is the extensive generalized form of the First Law. The fundamental dimensions of Equation j are:

$$[E] = \left(\frac{[m][L]^2}{[t]^2} \right) \left(\frac{[t]^2[F]}{[m][L]} \right) \left(\frac{[E]}{[F][L]} \right)$$

In the SI system, the units of equation j are:

$$J = (kg) \left(\frac{m^2}{s^2} \right) \left(\frac{s^2 N}{m kg} \right) \left(\frac{J}{N m} \right)$$

And in the USCS (English System), the units of j are:

$$Btu = (lb_m) \left(\frac{ft^2}{s^2} \right) \left(\frac{1}{32.2} \frac{s^2 lb_f}{ft lb_m} \right) \left(\frac{Btu}{778 lb_f ft} \right)$$

Note that the use of g_c is demanded in the equation for dimensional homogeneity.

Equation j can similarly represent unit mass of a system (shown by lower case symbols), independent of mass. The intensive form results from dividing by the system mass, m:

$$q = \frac{Q}{m}, \text{ etc.}$$

Yielding:

$$q_1^2 - w_1^2 = u_2 - u_1 + \frac{(V_2^2 - V_1^2)}{2} + g(z_2 - z_1) \quad k$$

Equation k is the generalized form of the First Law in intensive terms, that is, independent of mass of the system.

5.4. The First Law as a Rate Equation.

To obtain the differential equations for the closed system it is convenient to start with Equation h and divide by Δt since time is the independent variable.

$$\frac{\delta Q}{\Delta t} - \frac{\delta W}{\Delta t} = \frac{\Delta E}{\Delta t} \quad l$$

Then allowing Δt to approach zero, by definition of the derivative

$$\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \frac{dE}{dt} \quad m$$

In a common notation

$$\dot{Q} - \dot{W} = \frac{dE}{dt} \quad n$$

The time rate of change of heat and work must equal the time rate of change of energy within the system.

5.5. The First Law in Differential Form.

At times it is convenient to express the First Law in differential form, for instance, when involved in mathematical manipulations. Multiplying the above (Equation n) by an increment in time Δt yields

$$\frac{\delta Q}{dt} \Delta t - \frac{\delta W}{dt} \Delta t = \frac{dE}{dt} \Delta t \quad o$$

By definition of the differential of a dependent variable

$$dQ - dW = dE \quad p$$

Note that Q, W, and E are functions of a single independent variable of time.

Reverting back to the generalized form simply involves integration over time, from time t_1 to time t_2

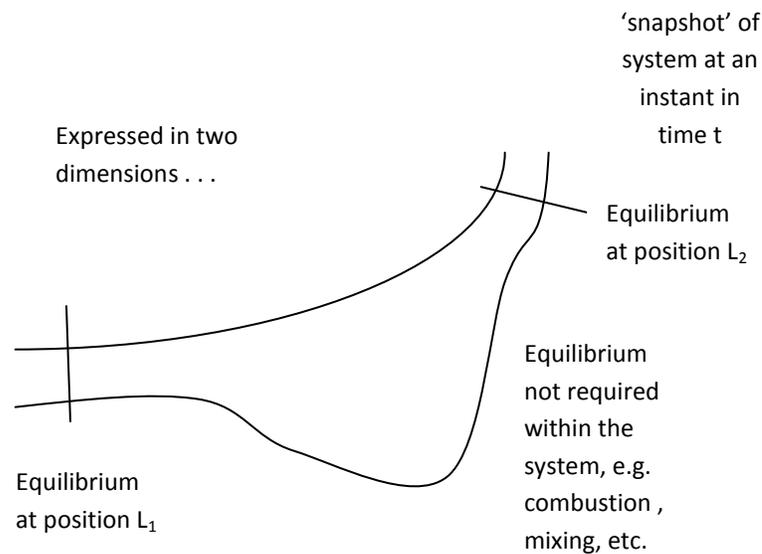
$$Q_1^2 - W_1^2 = \Delta E = U_2 - U_1 + \frac{m(V_2^2 - V_1^2)}{2} + mg(Z_2 - Z_1) \quad j$$

5.6. The First Law for the Open System.

The realm of classical thermodynamics includes the open systemⁱ. The system is a specified region of study where transfers of energy and mass are to be studied. The boundary of a system is the hypothetical envelope that surrounds the system.

As with the closed system, classical thermodynamics allows the study of a region where equilibrium may or may not exist as in the case of combustion, expansion of substances moving at random, chaotic mixing of substances, etc.

By defining the boundary of a system where equilibrium exists allows thermodynamic analysis of the entire system. Only the independent variable time is of concern in the closed system. In the open system the independent variables of location L and time t become essential for analysis.



Thus the engineer's concept of the dynamic open system allows analyses to be made of continuous streams which are far from equilibrium in part of their travels. *The procedure is to transform the problem from study of a fixed mass to study of a region, called by Prandtl a 'control volume'.* For the sake of parallelism the control volume is referred to as the 'open system' contrasting to earlier presentations of the 'closed system'.ⁱⁱ The phrase 'control volume' is often used in other thermodynamic presentations.

The energy of a flowing stream relative to a position coordinate L can be found by considering the moving element of mass Δm_f . Here the energy accompanying mass is made up of internal, kinetic and potential energies and the work done by the substance following Δm_f to push Δm_f through the distance $V\Delta t$ due to viscous forces (frictional forces are not considered).

Note that in the general sense $L=f(x, y, z)$ and the substance having mass m and properties of p , v , T , etc is thus defined in the independent variables of space L and time t .

Then,

$$\text{Energy of } \Delta m_f = \left(u + \frac{v^2}{2} + gZ \right) \Delta m_f$$

And,

$$\text{Work done on } \Delta m_f = \text{force} \times \text{distance} = pAV\Delta t = pv\Delta m_f$$

Here, the thermodynamic properties of p and v were selected judiciously because it is wise to express this work in terms of thermodynamic properties. This will become obvious.

With these relationships, the exact energy flow across L as $\Delta m_f = m_{f2} - m_{f1}$ crosses the coordinate L is

$$\Delta E_f|_L = \int_{m_{f1}(t1)}^{m_{f2}(t2)} \left(u + pv + \frac{v^2}{2} + gZ \right) dm_f$$

Comparison of the bracketed term to the *first law of the closed system* shows that the change in viewpoint from watching a fixed mass as it moves in space to watching a position in space has introduced a new energy term called the flow energy, pv . Note that the product pv is a property (a function of other thermodynamic properties 'p' and 'v' and has a nonzero value for both open and closed systems but takes on the significance of energy for a flow stream of the open system.

The energy pv is sometimes called 'flow work' since it originated from a work analysis. However, pv is a property, and work is not; hence the name of flow energy is more descriptive.

When the interior of the open system is analyzed, the procedure is essentially the same as for the closed system. For the stationary element of mass at time t is simply

$$e = u + \frac{v^2}{2} + gZ$$

And

$$E|_{system\ at\ time\ t} = \int_{L1}^{L2} e dm = \int_{L1}^{L2} e \rho A dL$$

Heat is energy in transition, each point on the surface of the system can be assigned a heat flux density. The heat transferred across the boundary of the system is

$$Q|_{t1}^{t2} = \int_{t1}^{t2} \int_{surface} \frac{\partial Q}{\partial t \partial A} dA dt$$

Where

$$heat\ rate\ \dot{Q} = \frac{\partial Q}{\partial t}$$

The heat rate here is expressed as a partial derivative because it is also a function of location L. In open system analysis we more often work with rates rather than heat quantity.

Similarly

$$W|_{t1}^{t2} = \int_{t1}^{t2} \int_{surface} \frac{\partial W}{\partial t \partial A} dA dt$$

Noting that in mechanical engineering, work is often expressed as shaft or moving boundary work such as a in piston-cylinder although work can also be that of pushing aside the atmosphere as in an inflating balloon.

Since energy is always conserved

$$-\Delta E|_{surroundings} = \Delta E|_{system}$$

Then by combining the above, and keeping with the convention of

- +heat and mass-energy to the system
- heat and mass-energy from the system
- +work from the system
- work to the system

We arrive at an integral form of the energy equation

$$\int_{t_1}^{t_2} \int_{surface} \frac{\partial^2 Q}{\partial t \partial A} dA dt - \int_{t_1}^{t_2} \int_{surface} \frac{\partial^2 W}{\partial t \partial A} dA dt + \int_{t_1}^{t_2} e_f dm_f|_{L_1} - \int_{t_1}^{t_2} e_f dm_f|_{L_2}$$

$$= \int_{L_1}^{L_2} e \rho A dL_{t_2} - \int_{L_1}^{L_2} e \rho A dL_{t_1}$$

The First Law for the open system (mass crosses the system boundaries) can be derived from the conservation of energy principle through simple mathematical elemental analysis. It is essential to note that the energy of flow, the product of p and v , are incorporated into the definition of enthalpy.

$$h = u + pv$$

And, unlike the closed system presentation, the velocity and potential energy of the flow must be accounted. Thus the First Law for the open system becomes:

$$\dot{Q}_{12} - \dot{W}_{12} = \frac{dE}{dt} \Big|_{system} + \sum_{i=1}^j \dot{m}_o \left(h_o + \frac{V_o^2}{2} + gz_o \right) - \sum_{i=1}^j \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) \quad (1)$$

where \dot{Q}_{12} and \dot{W}_{12} are the rate of heat and work transferred across the boundary in an increment in time from t_1 to t_2 and $\frac{dE}{dt} \Big|_{system}$ is the energy of the system (not of the flow).

And where

$$\frac{dE}{dt} \Big|_{system} = \frac{dU}{dt} + \frac{V^2}{2} + gz \quad (2)$$

once again of the system and not of the flow.

In the case of steady state, that is where the system is not changing as a function of time, the First Law reduces to:

$$\dot{Q}_{12} - \dot{W}_{12} = \sum_{i=1}^j \dot{m}_o \left(h_o + \frac{V_o^2}{2} + gz_o \right) - \sum_{i=1}^j \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) \quad (3)$$

In the case of steady flow (and to simplify further, one inlet and one outlet...):

$$\dot{Q}_{12} - \dot{W}_{12} = \dot{m} \left((h_o - h_i) + \frac{(V_o^2 - V_i^2)}{2} + g(z_o - z_i) \right) \quad (4)$$

Equation 4 above can be expressed a mass basis since $q = \frac{\dot{Q}}{\dot{m}}$, etc.

$$q - w = (h_o - h_i) + \frac{(V_o^2 - V_i^2)}{2} + g(z_o - z_i) \quad (5)$$

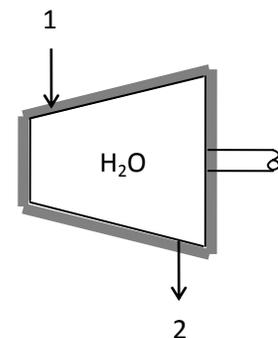
Equation 1 above should always be used as a starting point of an application during analysis of the open system and simplified though stated assumptions.

Example 1: Steam enters a turbine at 10 MPa and 500 C and leaves at 10 kPa with a quality of 90 percent. The process occurs rapidly and can be considered adiabatic.

Determine the mass flow rate required for a power output of 5 MW.

Assumptions:

1. The system is steady state.
2. The air flow is steady state.
3. The process occurs rapidly and is adiabatic.
4. Kinetic and potential energy of the air flow is negligible.



Solution:

From the steam tables

$$\left. \begin{array}{l} P_1 = 10 \text{ MPa} \\ T_1 = 500^\circ\text{C} \end{array} \right\} h_1 = 3375.1 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_2 = 10 \text{ kPa} \\ x_2 = 0.90 \end{array} \right\} h_2 = h_f + x_2 h_{fg} = 191.81 + 0.90 \times 2392.1 = 2344.7 \text{ kJ/kg}$$

$$\dot{Q}_{12} - \dot{W}_{12} = \frac{dE}{dt}_{system} + \sum_{i=1}^j \dot{m}_o \left(h_o + \frac{V_o^2}{2} + gz_o \right) - \sum_{i=1}^j \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right)$$

$$\dot{m}_o = \dot{m}_i = \dot{m} \text{ and } w = \frac{\dot{W}}{\dot{m}}$$

$$-w = (h_o - h_i) = 2344 \text{ kJ/kg} - 3375.1 \text{ kJ/kg} = -1031 \text{ kJ/kg}$$

$$w = 1031 \text{ kJ/kg (work out from the system is positive)}$$

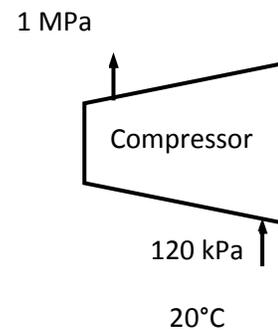
$$\dot{m} = \frac{\dot{W}}{w} = \frac{5000 \text{ kW}}{1031 \text{ kJ/kg}} \cdot \frac{\text{kJ/s}}{\text{kW}} = 4.852 \text{ kg/s}$$

Example 2: Air at 120 kPa and 20 C enters a compressor at a volumetric flow of 10 l/s and exits at 1 MPa and 300 C. The process occurs rapidly and can be considered adiabatic.

Determine the work required by the compressor in kJ/kg and the power required to drive the air compressor in kW.

Assumptions:

1. The system is steady state.
2. The air flow is steady state.
3. The process occurs rapidly and is adiabatic.
4. Kinetic and potential energy of the air flow is negligible.
5. Air can be considered an ideal gas with constant specific heats.



Solution:

The constant pressure specific heat of air at the average temperature of $(20+300)/2=160^\circ\text{C}=433$ K is $c_p = 1.018 \text{ kJ/kg}\cdot\text{K}$. The gas constant of air is $R = 0.287 \text{ kJ/kg}\cdot\text{K}$.

$$\dot{Q}_{12} - \dot{W}_{12} = \frac{dE}{dt}\bigg|_{\text{system}} + \sum_{i=1}^j \dot{m}_o \left(h_o + \frac{V_o^2}{2} + gz_o \right) - \sum_{i=1}^j \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right)$$

$$\dot{m}_o = \dot{m}_i = \dot{m} \text{ and } w = \frac{\dot{W}}{\dot{m}}$$

$$-w = (h_o - h_i) = c_p(T_2 - T_1) = (1.018 \text{ kJ/kg} \cdot \text{K})(300 - 20)\text{K} = 285 \text{ kJ/kg}$$

$$w = -285 \text{ kJ/kg (work into the system is negative)}$$

And

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273 \text{ K})}{120 \text{ kPa}} = 0.7008 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{0.010 \text{ m}^3/\text{s}}{0.7008 \text{ m}^3/\text{kg}} = 0.01427 \text{ kg/s}$$

$$\dot{W} = \dot{m}w = 0.01427 \text{ kg/s} \cdot -285 \text{ kJ/kg} = -4068 \text{ kW}$$

ⁱ In part, from Obert, E., Concepts of Thermodynamics, McGraw Hill, 1960.

ⁱⁱ **Ludwig Prandtl** (1875 –1953) a German scientist who pioneered the development of rigorous systematic mathematical analyses used as the foundations of aerodynamics.