ME211 Thermodynamics
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The First Law for the Open System

The First Law for the open system (mass crosses the system boundaries) can be derived from the conservation of energy principle through simple mathematical elemental analysis and can be shown to be:

$$\dot{Q}_{12} - \dot{W}_{12} = \left.\frac{dE}{dt}\right|_{system} + \sum_{i=1}^{j} \dot{m}_o \left(h_o + \frac{V_o^{\ 2}}{2} + g z_o \right) - \sum_{i=1}^{j} \dot{m}_i \left(h_i + \frac{V_i^{\ 2}}{2} + g z_i \right) \tag{1}$$

where $\dot{Q}_{12}$ and $\dot{W}_{12}$ are the rate of heat and work transferred across the boundary in an increment in time from t_1 to t_2 and $\left.\frac{dE}{dt}\right|_{system}$ is the <u>energy of the system</u> *(not of the flow)*.

And where

$$\left.\frac{dE}{dt}\right|_{system} = \frac{dU}{dt} + \frac{V^{\ 2}}{2} + gz \tag{2}$$

once again <u>of the system</u> and *not of the flow*.

In the case of steady state, that is where the system is not changing as a function of time, the First Law reduces to:

$$\dot{Q}_{12} - \dot{W}_{12} = \sum_{i=1}^{j} \dot{m}_o \left(h_o + \frac{V_o^{\ 2}}{2} + g z_o \right) - \sum_{i=1}^{j} \dot{m}_i \left(h_i + \frac{V_i^{\ 2}}{2} + g z_i \right) \tag{3}$$

In the case of steady flow (and to simplify further, one inlet and one outlet…):

$$\dot{Q}_{12} - \dot{W}_{12} = \dot{m} \left((h_o - h_i) + \frac{(V_o^2 - V_i^2)}{2} + g(z_o - z_i) \right) \tag{4}$$

Equation 4 above can be expressed a mass basis since $q = \frac{\dot{Q}}{\dot{m}}$, etc.

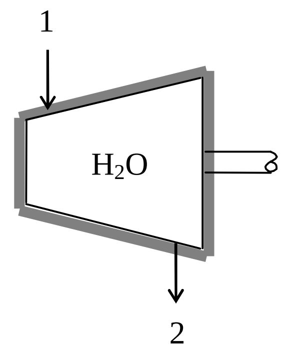
$$q - w = (h_o - h_i) + \frac{(V_o^2 - V_i^2)}{2} + g(z_o - z_i) \tag{5}$$

Equation 1 above should always be used as a starting point of an application and simplified though stated assumptions.

Example 1: Steam enters a turbine at 10 MPa and 500 C and leaves at
10 kPa with a quality of 90 percent. The process occurs rapidly and can be considered adiabatic.

Determine the mass flow rate required for a power output of 5 MW.

Assumptions:
1. The system is steady state.
2. The air flow is steady state.
3. The process occurs rapidly and is adiabatic.
4. Kinetic and potential energy of the air flow is negligible.



Solution:

From the steam tables

$$\left.\begin{array}{l} P_1 = 10 \text{ MPa} \\ T_1 = 500°\text{C} \end{array}\right\} h_1 = 3375.1 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 10 \text{ kPa} \\ x_2 = 0.90 \end{array}\right\} h_2 = h_f + x_2 h_{fg} = 191.81 + 0.90 \times 2392.1 = 2344.7 \text{ kJ/kg}$$

$$\dot{Q}_{12}^{\,0} - \dot{W}_{12} = \left.\frac{dE}{dt}\right|_{system}^{0} + \sum_{i=1}^{j} \dot{m}_o \left(h_o + \frac{V_o^2}{2}^{0} + g z_o^{\,0} \right) - \sum_{i=1}^{j} \dot{m}_i \left(h_i + \frac{V_i^2}{2}^{0} + g z_i^{\,0} \right)$$

$$\dot{m}_o = \dot{m}_i = \dot{m} \text{ and } w = \frac{\dot{W}}{\dot{m}}$$

$$-w = \left(h_o - h_i\right) = 2344 \text{ kJ/kg} - 3375.1 \text{ kJ/kg} = -1031 \text{ kJ/kg}$$

$w = 1031 \text{ kJ/kg}$ *(work out from the system is positive)*

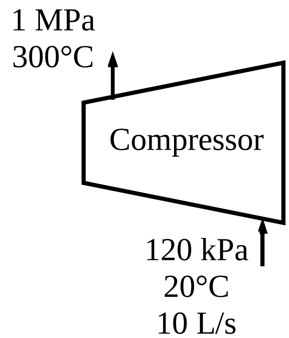
$$\dot{m} = \frac{\dot{W}}{w} = \frac{5000 \text{ kW}}{1031 \text{ kJ/kg}} \cdot \frac{\text{kJ/s}}{\text{kW}} = 4.852 \text{ kg/s}$$

Example 2: Air at 120 kPa and 20 C enters a compressor at a volumetric flow of 10 l/s and exits at 1 MPa and 300 C. The process occurs rapidly and can be considered adiabatic.

Determine the work required by the compressor in kJ/kg and the power required to drive the air compressor in kW.

Assumptions:

1. The system is steady state.
2. The air flow is steady state.
3. The process occurs rapidly and is adiabatic.
4. Kinetic and potential energy of the air flow is negligible.
5. Air can be considered an ideal gas with constant specific heats.



Solution:

The constant pressure specific heat of air at the average temperature of (20+300)/2=160°C=433 K is c_p = 1.018 kJ/kg·K. The gas constant of air is R = 0.287 kJ /kg·K.

$$\cancelto{0}{\dot{Q}_{12}} - \dot{W}_{12} = \cancelto{0}{\left.\frac{dE}{dt}\right|_{system}} + \sum_{i=1}^{j} \dot{m}_o\left(h_o + \cancelto{0}{\frac{V_o^2}{2}} + \cancelto{0}{gz_o}\right) - \sum_{i=1}^{j} \dot{m}_i\left(h_i + \cancelto{0}{\frac{V_i^2}{2}} + \cancelto{0}{gz_i}\right)$$

$$\dot{m}_o = \dot{m}_i = \dot{m} \text{ and } w = \frac{\dot{W}}{\dot{m}}$$

$$-w = \left(h_o - h_i\right) = c_p\left(T_2 - T_1\right) = (1.018\,\text{kJ/kg}\cdot\text{K})(300-20)\text{K} = 285\,\text{kJ/kg}$$

$w = -285\,\text{kJ/kg}$ *(work into the system is negative)*

And

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273\,\text{K})}{120\,\text{kPa}} = 0.7008\,\text{m}^3/\text{kg}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{0.010\,\text{m}^3/\text{s}}{0.7008\,\text{m}^3/\text{kg}} = 0.01427\,\text{kg/s}$$

$$\dot{W} = \dot{m}w = 0.01427\,\text{kg/s}\cdot -285\,\text{kJ/kg} = -4068\,\text{kW}$$