Generalized Form of the First Law

The general form of the First Law in expressed as a rate, derived previously, is:

$$\dot{Q}\Big|_{surface} - \dot{W}\Big|_{surface}$$
$$= \frac{dE}{dt}\Big|_{system} + \sum \dot{m}_o(h_o + \frac{V_o^2}{2} + gz_o)\Big|_{flow} - \sum \dot{m}_i(h_i + \frac{V_i^2}{2} + gz_i)\Big|_{flow}$$

As indicated in the equation above heat and work are surface phenomena and are path dependent. The independent variables are time and position. The rate of change within the system is given by the first term on the right hand side, dE/dt , and the two remaining terms, represented by the summation signs, represent the energy in flow in and out of the system.

Closed System Analysis

In the case where there is no flow crossing the system boundary (closed system), then $\dot{m}_i = 0$ and $\dot{m}_o = 0$. The flow terms thus drop leaving time as the only independent variable, since position is no longer relevant. The equation becomes the rate form of the First Law for the closed system (no mass transfer)

$$\dot{Q}\Big|_{surface} - \dot{W}\Big|_{surface} = \frac{dE}{dt}\Big|_{system}$$

Expanding the right hand side yields

$$= \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$

$$= \frac{dU}{dt} + \frac{d(m\frac{V^2}{2})}{dt} + \frac{d(mgz)}{dt}$$

Expanding the shorthand notation terms of $\dot{Q}$ and $\dot{W}$

$$\dot{Q} = \frac{\delta Q}{dt}$$

$$\dot{W} = \frac{\delta W}{dt}$$

The notations δQ and δW are, in mathematical phraseology, indefinite or inexact differentials. Integrating with respect to the independent variable time

$$\int_{t_1}^{t_2} \frac{\delta Q}{dt} dt - \int_{t_1}^{t_2} \frac{\delta W}{dt} dt = \int_{t_1}^{t_2} \frac{dU}{dt} dt + \int_{t_1}^{t_2} \frac{d(m\frac{V^2}{2})}{dt} dt + \int_{t_1}^{t_2} \frac{d(mgz)}{dt} dt$$

yields

$$Q_{1\to 2} - W_{1\to 2} = \Delta E = m(u_2 - u_1) + \frac{m({V_2}^2 - {V_1}^2)}{2} + mg(z_2 - z_1)$$

which is the familiar extensive form of the First Law for the closed system. In the above heat Q and work W are path dependent (the differentials are inexact), i.e., the values of heat and work depend upon how the heat and work are transferred to or from the system. The subscripts 1 and 2 are shorthand for t_1 and t_2 (or the states of the system at t_1 and t_2). The internal energy, u, is a thermodynamic property, a function of other thermodynamic properties (Zeroth Law) and determined by the thermodynamic state of the system. The velocity change and position change refer to that of the system.

Dividing the above by mass yields the intensive form of the First Law for the closed system,

$q = \frac{Q}{m}, w = \frac{W}{m}, u = \frac{U}{m}$, etc., yields

$$q_{1\to 2} - w_{1\to 2} = u_2 - u_1 + \frac{({V_2}^2 - {V_1}^2)}{2} + g(z_2 - z_1)$$

Simplified Open System* Analysis

Returning to the general form of the First Law in rate form

$$\dot{Q}\Big|_{surface} - \dot{W}\Big|_{surface} = \frac{dE}{dt}\Big|_{system} + \sum \dot{m}_o(h_o + \frac{V_o^2}{2} + gz_o)\Big|_{flow} - \sum \dot{m}_i(h_i + \frac{V_i^2}{2} + gz_i)\Big|_{flow}$$

For the case where there is only one inlet and one outlet

$$\dot{Q}\Big|_{surface} - \dot{W}\Big|_{surface} = \frac{dE}{dt}\Big|_{system} + \dot{m}_o(h_o + \frac{V_o^2}{2} + gz_o)\Big|_{flow} - \dot{m}_i(h_i + \frac{V_i^2}{2} + gz_i)\Big|_{flow}$$

In the case of steady state steady flow the transient term of the system drops and since

$$\dot{m}_o = \dot{m}_i = \dot{m}$$

* The term *open system* was first referred to as a *control volume* by **Ludwig Prandtl** (1875 –1953) a German scientist who pioneered the development of rigorous systematic mathematical analyses used as the foundations of aerodynamics. Many texts still refer to the control volume as the open system. The terms are synonymous.

Then

$$\dot{Q} - \dot{W} = \dot{m}\left[(h_o - h_i) + \frac{(V_o^2 - V_i^2)}{2} + g(z_o - z_i)\right]$$

Dividing by the mass flow rate

$$q = \frac{\dot{Q}}{\dot{m}}$$

$$w = \frac{\dot{W}}{\dot{m}}$$

yields the intensive form of the First Law for the open system

$$q - w = (h_o - h_i) + \frac{(V_o^2 - V_i^2)}{2} + g(z_o - z_i)$$

A Note about Heat and Work

The rate form of heat

$$\frac{\delta Q}{dt}$$

is the subject of heat transfer (which encompasses conduction, convection, and radiation).

The rate form of work can be subdivided into several types or subcategories

$$\frac{\delta W}{dt} = \left.\frac{\delta W}{dt}\right|_{boundary} + \left.\frac{\delta W}{dt}\right|_{mechanical} + \left.\frac{\delta W}{dt}\right|_{electrical}$$

Mechanical work is evidenced by a mechanical shaft that crosses the system boundary. Electrical work is evidenced by electrical leads crossing the system boundary. Boundary work is evidenced by the physical movement or displacement of the system boundary.

Boundary work to or from the system can be further reduced, or expressed, in terms of thermodynamic properties, usually p and V

$$\left.\frac{\delta W}{dt}\right|_{boundary} = p\frac{dV}{dt} = p\frac{d(mv)}{dt}$$

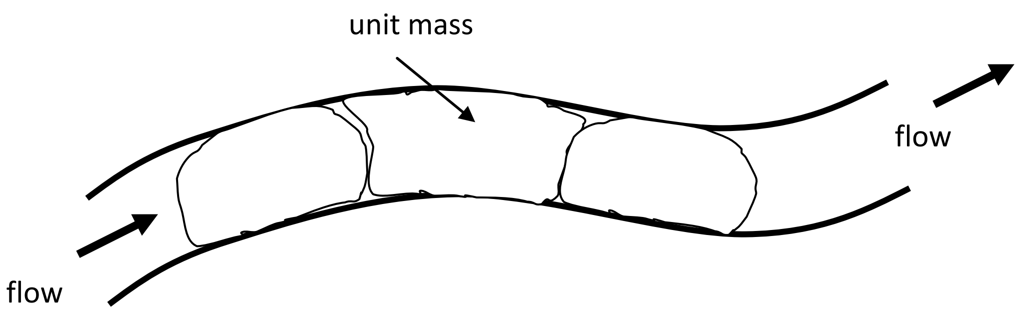
Or in integral form

$$W = \int_1^2 pdV$$

Work to or from a Flowing Fluid

Three different forms of work were presented in the text above: mechanical, electrical, and that due to a moving or deforming boundary. There are others which are less encountered in the thermosciences. However there is one more which frequently occurs when considering the open system; that is, work to or from a flowing gas or liquid.

Consider a unit mass of substance, infinitesimal in size, moving through a steady flow system. Mass does not cross the system boundary even though the substance is the same as that which surrounds it.



First law closed system analysis yields, in differential form

$$\delta q - \delta w = du + dKE + dPE$$

Even though mass does not cross the system boundary, the boundary of the unit mass deforms because of pressure and volume changes. The work then is that of a moving boundary

$$\delta w = du + dKE + dPE - \delta q$$

$$-pdv = du + dKE + dPE - \delta q$$

since

$$\delta w|_{boundary,closed\ system} = pdv$$

Now consider the same unit of mass <u>fixed in location</u> as a substance moves across the system boundary in a steady state situation. First law open system analysis yields, in differential form

$$-\delta w|_{steady\ flow} = dh + dKE + dPE$$

Since by definition

$$h = u + pv$$

then by differentiating the above

$$dh = du + d(pv) = du + pdv + vdp$$

And

$$-\delta w|_{steady\ flow} = du + pdv + vdp + dKE + dPE - \delta q$$

Simple rearrangement

$$-\delta w|_{steady\ flow} = pdv + vdp + (du + dKE + dPE - \delta q)$$

The last four terms on the right hand side are equal to the boundary work *pdv* illustrated above.

Upon substitution

$$-\delta w|_{steady\ flow} = pdv + vdp + (-pdv)$$

Thus

$$\delta w|_{steady\ flow} = -vdp$$

Upon integration with position and time as the independent variables (the open system)

$$\int \delta w|_{steady\ flow} = \int -vdp$$

Yields

$$w|_{steady\ flow,i\to o} = \int -vdp$$

Where *i* is the inlet, *o* is the outlet, *v* is the specific volume of the fluid, and *p* is the pressure of the fluid. In extensive form, easily derived through parallel development

$$W|_{steady\ flow,i\to o} = \dot{m}\int -vdp$$

To reiterate, the work to/from a substance within a closed system is

$$\delta w|_{boundary,closed\ system} = pdv$$

The work to/from a substance in a steady state steady flow open system is

$$w|_{steady\ flow,i\to o} = \int -vdp$$

There are more eloquent and mathematically pleasing methods to derive work for the steady state open system. These methods utilize entropy, the definition of the differential, etc., and are not presented here. The reader is referred to any one of a number of thermodynamic texts for other derivation methods.

Note that closed system work is shown as the area underneath the pV diagram while open system work in steady flow is shown as the negative of the area *to the left* of the pV diagram. The above equation is particularly useful when calculating the work to/from an incompressible liquid (such as water) because the specific volume v does not change significantly with changes in pressure. Therefore, for an incompressible liquid

$$w|_{steady\ flow, i \to o} = -v \int dp \approx v\Delta p = v(p_2 - p_1)$$

Consider the following illustrative and simple example. Water is pumped from a storage tank at atmospheric conditions, 100 kPa, 20 C, to 1500 kPa, 20C. Here, water remains a liquid in the pumping process (a simple water pump supplying water for residential use) and because water is nearly incompressible, the specific volume is constant, v = 01001 m^3/kg. Thus

$$w|_{steady\ flow, i \to o} = v(p_2 - p_1) = \ 0.001\ \frac{m^3}{kg}(1500 - 100)kPa\left(\frac{N}{m^2 Pa}\right)\left(\frac{J}{Nm}\right)\left(\frac{kJ}{1000J}\right)$$

$$w = 0.0014\frac{kJ}{kg}$$

If the mass flow rate of the water is 5 kg/s, then

$$W = 5\frac{kg}{s} \times 0.0014\frac{kJ}{kg} = 0.007\frac{kJ}{s}\left(\frac{kWs}{kJ}\right) = 0.007kW$$

Note that it is relatively easy to compress an incompressible substance. This is not true of compressible substances where the specific volume varies with pressure (such as that encountered with an ideal or real gas). In this case, the function *v = f(p,T)* must be known before the integration can be evaluated.