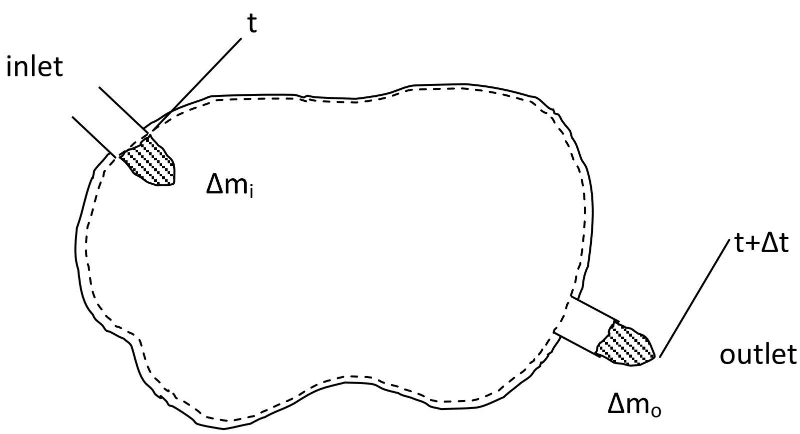
Derivation of the First Law for the Open System

Consider the following system:



Energy of the incremental flow Δm

$$E|_{t+\Delta t} = \ energy\ of\ \Delta m\ at\ time\ t + \Delta t$$

Energy of the system at time t

$$E_1 = E_t + e_i \Delta m_i$$

Energy of the system at time t+Δt

$$E_2 = E_{t+\Delta t} + e_o \Delta m_o$$

Then

$$E_2 - E_1 = E_{t+\Delta t} + e_o \Delta m_o - E_t - e_i \Delta m_i$$

Or rearranging

$$E_2 - E_1 = E_{t+\Delta t} - E_t + e_o \Delta m_o - e_i \Delta m_i$$

Dividing by an increment in time

$$\frac{E_2 - E_1}{\Delta t} = \frac{E_{t+\Delta t} - E_t}{\Delta t} + e_o \frac{\Delta m_o}{\Delta t} - e_i \frac{\Delta m_i}{\Delta t} \qquad (1)$$

Now consider the work required to move an increment of flow Δm_i and Δm_o

$$\delta W|_{flow} = -\sigma_n A \Delta l = -\sigma_n v \Delta m = pv\Delta m$$

Where σ_n is the normal shear force of the fluid.

Total work of the control volume

$$\delta W = \delta W|_{cv} + \delta W|_{flow}$$

$$\delta W = \delta W|_{cv} + pv\Delta m$$

$$\frac{\delta W}{\Delta t} = \left.\frac{\delta W}{\Delta t}\right|_{cv} + p_i v_i \frac{\Delta m_i}{\Delta t} - p_o v_o \frac{\Delta m_o}{\Delta t} \qquad (2)$$

First law

$$\delta Q - \delta W = E_2 - E_1$$

Divide by Δt

$$\frac{\delta Q}{\Delta t} - \frac{\delta W}{\Delta t} = \frac{E_2 - E_1}{\Delta t} \qquad (3)$$

Substituting (1) and (2) into (3)

$$\left.\frac{\delta Q}{\Delta t}\right|_{cv} - \left.\frac{\delta W}{\Delta t}\right|_{cv} = \frac{E_{t+\Delta t} - E_t}{\Delta t} + \frac{\Delta m_o}{\Delta t}(e_o + p_o v_o) - \frac{\Delta m_i}{\Delta t}(e_i + p_i v_i)$$

Noting also

$$e = u + \frac{V^2}{2} + gz$$

Then

$$\left.\frac{\delta Q}{\Delta t}\right|_{cv} - \left.\frac{\delta W}{\Delta t}\right|_{cv} = \frac{E_{t+\Delta t} - E_t}{\Delta t} + \frac{\Delta m_o}{\Delta t}\left(u_o + p_o v_o + \frac{V_o{}^2}{2} + gz_o\right) - \frac{\Delta m_i}{\Delta t}\left(u_i + p_i v_i + \frac{V_i{}^2}{2} + gz_i\right)$$

And using the definition of enthalpy

$$h = u + pv$$

$$\lim_{\Delta t \to 0} \left. \frac{\delta Q}{\Delta t} \right|_{cv} - \left. \frac{\delta W}{\Delta t} \right|_{cv} = \frac{E_{t+\Delta t} - E_t}{\Delta t} + \frac{\Delta m_o}{\Delta t}\left(h_o + \frac{{V_o}^2}{2} + gz_o\right) - \frac{\Delta m_i}{\Delta t}\left(h_i + \frac{{V_i}^2}{2} + gz_i\right)$$

$$\dot{Q} - \dot{W} = \left. \frac{dE}{dt} \right|_{system} + \dot{m}_o\left(h_o + \frac{{V_o}^2}{2} + gz_o\right) - \dot{m}_\iota\left(h_i + \frac{{V_i}^2}{2} + gz_i\right)$$

Where

$$\dot{m} = \rho A \vec{V}$$

Or in the case of multiple inlets and outlets we have the more general case

$$\dot{Q} - \dot{W} = \left. \frac{dE}{dt} \right|_{system} + \sum \dot{m}_o\left(h_o + \frac{{V_o}^2}{2} + gz_o\right) - \sum \dot{m}_\iota\left(h_i + \frac{{V_i}^2}{2} + gz_i\right)$$

In the case of steady state steady flow

$$\left. \frac{dE}{dt} \right|_{system} = 0$$

And

$$\dot{m}_\iota = \dot{m}_o = \dot{m}$$

Then

$$\dot{Q} - \dot{W} = \dot{m}\left(h_o - h_i + \frac{{V_o}^2 - {V_i}^2}{2} + g(z_o - z_i)\right)$$

On an intensive basis

$$w = \frac{\dot{W}}{\dot{m}}$$

$$q = \frac{\dot{Q}}{\dot{m}}$$

$$q - w = \dot{m}\left(h_o - h_i + \frac{{V_o}^2 - {V_i}^2}{2} + g(z_o - z_i)\right)$$