ME 211 and ME312 Thermodynamics Equation Sheet

D. Abata, April 1, 2020

Conservation of mass: $\left.\frac{dm}{dt}\right|_{system} = \sum_{i=1}^{k} \dot{m}_o - \sum_{o=1}^{j} \dot{m}_1$ where $\dot{m} = \rho A V$

<u>Boundary</u> work any system: $W|_{bndry} = \int_1^2 pdV$ and flow work (open system) $W|_{flow} = -\int_1^2 vdp - \Delta ke - \Delta pe$

$W|_{T=C} = mRT\left(ln\frac{V_2}{V_1}\right)$, assuming ideal gas and since T=C then $p_1V_1 = p_2V_2 = p_nV_n$ and $\frac{p_1}{p_2} = \frac{V_2}{V_1}$

$W|_{p=C} = p(V_2 - V_1)$

For the polytropic process, that is $pV^n = C$: $W = \int pdV = \int \frac{C}{V^n} dV = \frac{p_2V_2 - p_1V_1}{1-n} = \left.\frac{mR(T_2-T_1)}{1-n}\right|_{ideal\ gas}$

<u>Open system</u> work: $\left.\dot{W}\right|_{open} = -\dot{m}\int_1^2 vdp$, $\left.\dot{W}\right|_{poly} = -\frac{\dot{m}nR(T_2-T_1)}{n-1}$, $\left.\dot{W}\right|_{isothermal} = -\dot{m}RT\frac{p_2}{p_1}$

Through application of the differential $u = f(T, v)$ and $h = f(T, p)$, then: $c_v = \left.\frac{\partial u}{\partial T}\right|_{v=C}$ and $c_p = \left.\frac{\partial h}{\partial T}\right|_{p=C}$

Ideal gas: $U = f(T)|_{alone}$, $pV = nR_oT$, $R = \frac{R_o}{MW}$, $pV = mRT$, $pv = RT$, $c_v = \frac{du}{dT}$, $c_p = \frac{dh}{dT}$, $R = c_p - c_v$

$\Delta u = c_v(T_2 - T_1)$, $\Delta h = c_p(T_2 - T_1)$

Quality in the mixture region: $v = (1-x)v_f + xv_g$, $x = \frac{v - v_f}{v_g - v_f}$ *Note: v is interchangeable with u, h, or s*

For the closed system: $Q - W = \Delta E = \Delta U + m\frac{V^2}{2} + mg(z_2 - z_1)$

For the open system: $\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right|_{system} + \sum_{o=1}^{j} \dot{m}_o\left(h_o + \frac{V_o^2}{2} + gz_o\right) - \sum_{i=1}^{k} \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)$

where: $\left.\frac{dE}{dt}\right|_{system} = \frac{dU}{dt} + m\frac{dV^2}{2} + mgdz$

Thermal Efficiency, COP, and the Carnot cycle: $\eta|_t = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} = \frac{W_{net}}{Q_H}$, $COP = \frac{Q_L}{W_{net,in}} = \frac{Q_L}{Q_H - Q_L}$

$\eta|_{t,\ Carnot} = \frac{T_H - T_L}{T_H}$

Second Law Considerations:

$\Delta S = \int \frac{\partial Q}{T}$, $\Delta S = \left.\frac{Q}{T}\right|_{T=C}$, $\Delta S|_{net} = \Delta S|_{system} + \Delta S|_{surroundings} \geq 0$

For the ideal gas:

$\Delta s = c_v\ ln\frac{T_2}{T_1} + R\ ln\frac{V_2}{V_1}$, $\Delta s = c_p\ ln\frac{T_2}{T_1} - R\ ln\frac{p_2}{p_1}$

The above for s=0 and constant heat capacities: $\left.\frac{T_2}{T_1}\right|_{s=C} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{v_1}{v_2}\right)^{k-1}$ and $\left.\frac{p_2}{p_1}\right|_{s=C} = \left(\frac{v_1}{v_2}\right)^k$

For solids and to approximate compressed liquids: $\Delta s = c\ ln\frac{T_2}{T_1}$

Turbine and compressor efficiencies

$\eta_T = \frac{W_{act}}{W_{ideal}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$, $\eta_C = \frac{W_{ideal}}{W_{act}} = \frac{h_{2s} - h_1}{h_2 - h_1}$

- **PLEASE DO NOT WRITE ON THIS EQUATION SHEET.**
- **HAND IN THIS EQUATION SHEET WITH YOUR EXAM.**
- **THIS EQUATION SHEET WILL BE REUSED!**

Exergy

Nonflow exergy (intensive): $\phi = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$

Flow exergy (intensive): $\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$

Nonflow exergy change (intensive): $\Delta\phi = (u_2 - u_1) + p_0(v_2 - v_1) - T_0(s_2 - s_1) + \frac{{V_2}^2 - {V_1}^2}{2} + g(z_2 - z_1)$

Flow exergy change (intensive): $\Delta\psi = (h_2 - h_1) - T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$

Exergy transfer by heat: $\chi_{heat} = \left(1 - \frac{T_o}{T}\right) Q$

Exergy transfer by work (boundary work): $\chi_{work} = W - W_{surr} = W - p_o(v - v_o)$

Exergy transfer by work (other forms of work): $\chi_{work} = W$

Irreversibility: $I = \mathrm{X_{destroyed}} = \left(|\Delta \mathrm{X}|_{\mathrm{system}} - \mathrm{W_{useful,out}}\right) + \mathrm{X_{heat\ lost}}$

Second Law Efficiency (Effectiveness): $\eta_{II} = \frac{|W_{useful}|}{|X_{system}|}$

Gas Mixtures

Mass and mole fraction: $mf_i = \frac{m_i}{m_m}$ and $y_i = \frac{N_i}{N_m}$

Apparent or average molar mass and gas constant: $M_m = \frac{m_m}{N_m} = \sum_{i=1}^{k} y_i M_i$ and $R_m = \frac{R_u}{M_m}$

Dalton and Amagat: $p_m = \sum_{i=1}^{k} p_i$ and $V_m = \sum_{i=1}^{k} V_i$ Thus: $\frac{p_i}{p_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i$

Extensive properties (U, H, S): $U_m = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i$

Intensive properties (u, h, s, c_v, c_p): $\bar{c}_v = \sum_{i=1}^{k} y_i \bar{c}_{v,i}$ and $c_v = \sum_{i=1}^{k} mf_i c_{v,i}$

Gas-Vapor Mixtures (Psychrometrics)

$\omega = \frac{m_v}{m_a} = \frac{0.622 P_v}{P - P_v} = \frac{0.622 \phi P_g}{P - \phi P_g}$ where $P_g = P_{sat@T}$

$\phi = \frac{m_v}{m_g} = \frac{P_v}{P_g}$

$h = h_a + \omega h_g$

$\omega_1 = \frac{c_P(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g1} - h_{f2}}$

Combustion

$AF = \frac{m_{air}}{m_{fuel}}$, $Q - W = \sum N_p\, (\bar{h}_f^0 + \bar{h} - \bar{h}^0)_p - \sum N_r\, (\bar{h}_f^0 + \bar{h} - \bar{h}^0)_r$

Compressible Flow

$h_o = h + \frac{V^2}{2}$, $T_o = T + \frac{V^2}{2c_p}$, $\frac{p_o}{p} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$, $\frac{\rho_o}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$, $c = \sqrt{kRT}$, $Ma = \frac{V}{c}$

- **PLEASE DO NOT WRITE ON THIS EQUATION SHEET.**
- **HAND IN THIS EQUATION SHEET WITH YOUR EXAM.**
- **THIS EQUATION SHEET WILL BE REUSED!**