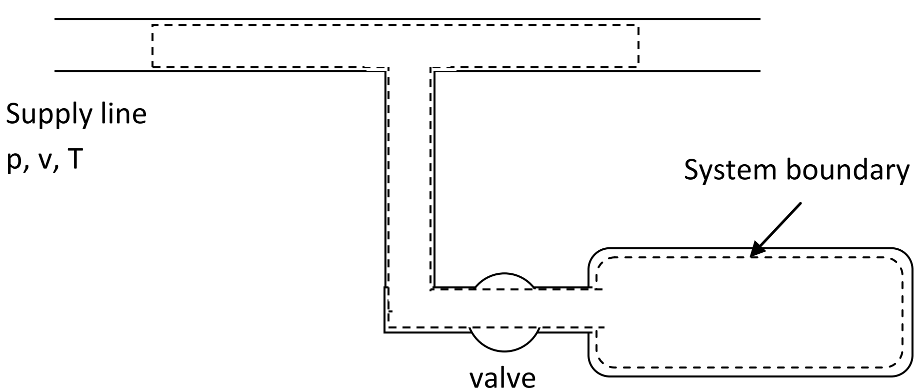
Transient Flow Analysis

1. The Filling Process
2. Discharging of a Cylinder (emptying a cylinder)
3. Discharge of an Ideal Gas

In the discussion on the first law for the open system applied to flow processes such as nozzles, pumps, turbines, heat exchangers, etc., emphasis has been placed on steady state steady flow (sssf) through these devices. With sssf the change of energy with respect to time, dE/dt, of the system does not change, mass flow is constant, and the properties of the fluid at a given location are invariant with respect to time. While these assumptions cover many applications of interest to the engineer, there are several applications involving unsteady flow which cannot be analyzed with sssf assumptions. For instance in the filling or emptying of a gas cylinder the properties of the fluid within the system vary with time and there is an accumulation of mass within the cylinder.

1. The Filling Process

Consider the following example which is a common encounter in engineering situations. A gas cylinder is charged from a supply line:



First, as always, the First Law, assuming no boundary work, no outflow, and negligible velocities of both the system and inlet flow:

$$\dot{Q} - \dot{W} = \frac{dE}{dt}\bigg|_{system} + \sum \dot{m}_o \left(h_o + \frac{V_o{}^2}{2} + gz_o\right) - \sum \dot{m}_\iota \left(h_i + \frac{V_i{}^2}{2} + gz_i\right)$$

$$\frac{dE}{dt} = \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$

And integrating the mass flow, heat transfer, and system internal energy over the time required to fill the tank, Δt:

$$\int_0^t \dot{m}\, dt = \int_0^t \frac{dm}{dt} dt = m_f - m_i$$

$$\int_0^t \dot{Q} dt = \int_0^t \frac{dQ}{dt} dt = Q_{process}$$

$$\int_0^t \frac{dU}{dt} dt = U_f - U_i = m_f u_f - m_i u_i$$

The first law becomes

$$Q + (m_f - m_i) h_f = m_f u_f - m_i u_i$$

If the tank is initially evacuated and the process occurs rapidly (without time for heat transfer), the solution is simplified,

$$h_f = u_f$$

$$c_p T_f = c_v T_f$$

$$T_f = \frac{c_p}{c_v} T_f$$

Which indicates the temperature of the air within the tank will not be same as the temperature of the supply line, instead it will be higher since the ratio of heat capacities, c_p/c_v , is always greater than one.

Example 1. Air at 5000 kPa and 300 K is flowing through a pipeline. An evacuated and insulated cylinder of volume 0.1 m3 is connected to the pipeline through a valve. The valve is opened and the cylinder is filled with air till the pressure in the cylinder reaches the line pressure. The valve is then closed. Assuming that the air behaves like an ideal gas with k = 1.4, determine the temperature of the air in the cylinder at the end of the filling operation and the mass of air that is filled in the cylinder.

Solution:

$$\dot{Q} - \dot{W} = \frac{dE}{dt}\bigg|_{system} + \sum \dot{m}_o \left(h_o + \frac{V_o{}^2}{2} + gz_o\right) - \sum \dot{m}_\iota \left(h_i + \frac{V_i{}^2}{2} + gz_i\right)$$

$Q + (m_f - m_i)h_f = m_f u_f - m_i u_i$

T_f = k X T = 1.4 X 300 = 420 K

m = pV/RT = 5000 X 0.1/0.2865 X 420 = 4.15 kg

Example 2. Steam at a pressure of 2000 kPa and 500 C is flowing in a pipe. An evacuated tank is connected to this pipe through a valve. The valve is opened and the tank is filled with steam until the pressure is 2000 kPa (line pressure), and then the valve is closed. The process takes place adiabatically and the kinetic energy and potential energy changes can be assumed negligible. Determine the temperature of the steam in the tank at the end of the filling operation.

$$\dot{Q} - \dot{W} = \frac{dE}{dt}\bigg|_{system} + \sum \dot{m}_o\left(h_o + \frac{V_o^2}{2} + gz_o\right) - \sum \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)$$

$$Q + (m_f - m_i)h_f = m_f u_f - m_i u_i$$

Thus, since the tank was initially empty *(evacuated)*

$$u_{tank} = h_f$$

From the Steam tables at T = 500 C, p = 2000 kPa

h = 3467 kJ/kg

Then, guess a temperature for u and iterate to 3467 kJ/kg. . .

at T= 600 C, u = 3290 kJ/kg

at T= 700 C, u = 3470 kJ/kg

By interpolation we find u = 3689 kJ/kg at 698 C

Example 3. A 2 m^3 tank with perfectly insulated walls contains saturated steam at a pressure of 1 MPa. This tank is connected through a valve to a steam line through which flows superheated steam at a pressure of 4 MPa and 400C. The valve is opened and steam is admitted rapidly into the tank until the pressure in the tank is 4 MPa. Estimate the mass of steam that enters the tank.

$$\dot{Q} - \dot{W} = \frac{dE}{dt}\bigg|_{system} + \sum \dot{m}_o\left(h_o + \frac{V_o^2}{2} + gz_o\right) - \sum \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)$$

$$Q + (m_f - m_i)h_f = m_f u_f - m_i u_i$$

(mf-m0) hi = $m_f u_f – m_o u_o$

Or rearranging

$m_f (h_f – u_f) = m_o (h_i – u_o)$

At x=1 and 1 MPa u_o = 2582 kJ/kg and v0 = 0.1943 m^3/kg

The initial mass of steam in the tank is

$m_o = V/v_o$ = 2 m^3 /1943 m^3/kg = 10.293 kg

The enthalpy of the steam in the line at 4MPa and 400 C from the tables is h_i = 3216 kJ/kg.

This is a trial and error solution since we do not know the mass or the final temperature. Assume the final temperature to be 425 C. Then from the steam tables

h_f = 3273 kJ/kg, u_f = 2967 kJ/kg, and v_f = 0.0766 m3/kg

Then mf = 2 m^3/0.0766 m^3/kg = 26.11 kg

And

Mf(3216 – u_f) = 10.293(3216 – 2582)

Which yields u_f = 2966 kJ/kg – the correct value of u_f at 425 C. The assumption of 425 C was correct.

The mass of steam that enters the tank is then:

mf – m_i = 26.11 – 10.293 = 15.817 kg.

2. Discharging of a Cylinder (emptying a cylinder)

Consider a cylinder containing a gas at high pressure. If the valve is opened, the gas inside the cylinder rushes out into the surroundings. During the process of discharging the tank, the rate at which the gas escapes from the cylinder, its condition and the condition of the gas inside the cylinder vary with time.

$$\dot{Q} = \frac{dE}{dt} + \dot{m}_o(h_o + \frac{V^2}{2})$$

$$\dot{m}_o = -\frac{dm}{dt}$$

$$\int_0^t \frac{dm}{dt} dt = m_f - m_o$$

$Q = (m_f u_f - m_o u_o)+(m_o - m_f)(h+V^2/2)$

3. Discharge of an Ideal Gas

Consider the case of a tank emptying air, an ideal gas, with no heat transfer, now work and negligible velocities. The system (the tank) has no associated velocity or potential energy either. Then the first law

$$\dot{Q} - \dot{W} = \frac{dE}{dt}\Big|_{system} + \sum \dot{m}_o \left(h_o + \frac{V_o^2}{2} + gz_o\right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right)$$

reduces to:

$$\dot{m}_o h_o = -\frac{dE}{dt} = \frac{d(mu)}{dt}$$

For the ideal gas:

$h = c_p T$ and $u = c_v T$

Then

$$c_p T \frac{dm}{dt} = \frac{d(mc_v T)}{dt}$$

Assume the temperature of the gas which escapes is the same as the temperature of the gas in the tank at any instant in time. Also note that cv is not a function of time and the product of m X T can be expanded in the differential,

$$T \frac{c_p}{c_v} \frac{dm}{dt} = m \frac{dT}{dt} + T \frac{dm}{dt}$$

Also, $k = c_p/c_v$, and with some minor algebra,

$$(k-1) T \frac{dm}{dt} = m \frac{dT}{dt}$$

$$\frac{(k-1)dm}{mdt} = \frac{dT}{Tdt}$$

Integrating over time t,

$$(k-1) \ln\left(\frac{m_f}{m_o}\right) = \ln\left(\frac{T_f}{T_o}\right)$$

Since the volume of the tank remains constant,

$m_o = p_o V/RT_o$ and $m_f = p_f V/RT_f$

then

$$\ln\left(\frac{p_f T_o}{p_o T_f}\right) = \frac{1}{1-k}\ln\left(\frac{T_f}{T_o}\right)$$

Or, assuming $T_f = T_o$

$$ln\frac{p_f}{p_o} = \frac{1}{1-k}\ln\left(\frac{T_f}{T_o}\right)$$

Or,

$$\frac{T_f}{T_o} = \left(\frac{p_f}{p_o}\right)^{\frac{k-1}{k}}$$

which is identical as an ideal gas going through a reversible adiabatic process.

Example 4. An insulated gas cylinder of volume 0.1 m^3 contains air (an ideal gas) at 5000 kPa and 300 K. The valve of the cylinder is opened allowing the air to escape till the air pressure in the cylinder reaches 3000 kPa. Determine the temperature of the air left in the cylinder and the mass of the air that escaped from the cylinder.

When an ideal gas discharges from an insulated tank, the gas remaining in the tank can be considered to have undergone a reversible adiabatic expansion from p_o, t_o to p_f, T_f.

$T_f/To = (p_f/p_o)^{k-1/k}$

$T_f = 300\ (3000/5000)^{0.28} = 259$ K

The initial mass of air in the cylinder is

m_o = pV/RT = (5000 kPa X 0.1 m3)/(0.287 kJ/kgK X 300 K) = 5.8 kg *units check* $\frac{kPa\left(\frac{kN}{kPa\ m^2}\right)m^3}{\frac{kJ}{kgK}\frac{kN\ m}{kJ}K}$

The final mass of air in the cylinder is

m = pV/RT = (3000 kPa X 0.1 m3)/(0.287 kJ/kgK X 259 K) = 4.0 kg

The mass of air that has escaped is

$m_o - m_f = 1.8$ kg

The examples above are from An Introduction to Thermodynamics by Y. V. C. Rao.

http://books.google.com/books?id=iYWiCXziWsEC&pg=PA164&lpg=PA164&dq=thermodynamics+filling&source=bl&ots=77KCzMVTyp&sig=DjGsApeTUEb4RRQxApAryOfmPFs&hl=en&sa=X&ei=la9nT8ppxemxAt_AoK4J&sqi=2&ved=0CEsQ6AEwBg#v=onepage&q=thermodynamics%20filling&f=false