The Thermodynamics of

Emptying a Pressure Vessel

A 30 gallon (0.1135 m^3) tank is filled with air (k = 1.4) at 150 psia (1034 kPa) and 70 ^oF (294 K). A valve connected to a pipe with an inside diameter of ½ inch (0.0127 m) is suddenly opened venting the air to the atmosphere at 14.7 psia (101 kPa).

Note that the numerical solution to the emptying process starts as always with the first law

$$\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right|_{system} + \sum \dot{m}_o \left(h_o + \frac{{V_o}^2}{2} + gz_o\right) - \sum \dot{m}_\iota \left(h_i + \frac{{V_i}^2}{2} + gz_i\right)$$

resulting with

$$\frac{c_v(m_{t+\Delta t}T_{t+\Delta t} - m_t T_t)}{\Delta t} = -\dot{m}_e(c_p T_e) = \; -\dot{m}_e(c_p T_t)$$

The test for choked flow is given by

$$\frac{p_T}{p_0} \le \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

where T_0 and p_0 are the temperature and pressure of the tank (stagnation values), p_T is the pressure at the throat of the orifice, k is the ratio of specific heats, and A_R is the effective flow area.

For choked flow

$$\dot{m}_e = \frac{A_R p_0}{R{T_0}^{\frac{1}{2}}} k^{\frac{1}{2}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

If the flow is not choked, then,

$$\dot{m}_e = \frac{A_R p_0}{R{T_0}^{\frac{1}{2}}} \left(\frac{p_T}{p_0}\right)^{\frac{1}{k}} \left\{\frac{2k}{k-1}\left[1 - \left(\frac{p_T}{p_0}\right)^{\frac{k-1}{k}}\right]\right\}^{\frac{1}{2}}$$

Plot the temperature and pressure of the air in the tank as a function of time and determine the time required to empty the tank. Note pressure in the tank is obtained through simple application of the ideal gas law $p_o = \frac{mRT_0}{V}$ where T_0 is the temperature of the air in the tank.

Note: It is best to use SI units

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%
%The Emptying Process
%
%The data generated by this program represents the following assumptions:
% 1.  exit velocity is negligible
% 2.  no heat transfer
% 3.  ideal gas model (should use Beattie Bridgeman) which is not
%     representative at lower temperatures.
%
%  October 4, 2017
%
P = zeros(400,1);
T = zeros(400,1);
m = zeros(400,1);
t = zeros(400,1);
choked = zeros(400,1);
R = 0.2865;
K = 1.4;
cp = 1.0035;
cv = 0.718;
P(1) = 1034.0;
Pe = 101.0;
T(1) = 294.0;
V = 0.1135;
D = 0.0127;
A = 3.14*D*D/4;
t(1) = 0.0;
delt = 0.5;
CRIT = (2/(K+1))^(K/(K-1));
% when K=1.4 CRIT=0.528

for n=1:400
m(n) = (P(n)*V)/(R*T(n));

if (Pe/P(n)) <= CRIT
% choked flow
mflow = (A*P(n)/(R*T(n)^0.5))*(K^0.5)*(2/(K+1))^((K+1)/(2*(K-1)));
choked(n) = 1;
else
% flow is not choked
mflow = (A*P(n)/(R*T(n)^0.5))*((Pe/P(n))^1/K)* (((2*K)/(K-1))*((1- ((Pe/P(n))^(K-1)/K))))^0.5;
choked (n)= 0;
end
% pressure of tank reaches atmospheric pressure
if P(n)< Pe + 5.
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    m(n+1)= m(n);
    T(n+1)= T(n);
    P(n+1)= Pe;
    mflow = 0.0;
else
m(n+1) = m(n) - mflow*delt;
T(n+1) = ((-mflow*delt*cp*T(n))/cv+m(n)*T(n))/m(n+1);
P(n+1) = m(n+1)*R*T(n+1)/V;
end
t(n+1) = t(n) + delt;

end
%output results

fileID=fopen('output.txt','w');
for n = 1:400
fprintf(fileID,'%5.0f  \t  %4.1f  \t  %4.1f  \t  %4.1f  \t  %2.1f  \t  %4.0f  \t  %6.3f \n', n, t(n), T(n),
P(n), Pe, choked(n), m(n));
end
fclose(fileID);
```