

Useful Equations

Displacement Volume, Compression Ratio, and Mean Piston Speed

$$V_d = \frac{\pi}{4} B^2 \cdot S \cdot n_{cyl} \quad \varepsilon \equiv \frac{V_{max}}{V_{min}} = \frac{V_c + V_d}{V_c} \quad S_p \equiv \frac{2 \cdot S \cdot N}{60}$$

Work, Power, and Efficiencies

Work (per revolution) $W = 2\pi T$

Power in kW (N in rpm, all other units are SI) $P = \frac{W}{t} = \frac{2\pi TN}{60000} = \frac{mep \cdot V_d \cdot N}{60 \cdot x}$

Indicated Power $IP = BP + FP$

Power in kW (N in rpm, Hu in kJ/kg, all other units are SI)

$$IP = \frac{\eta_v p_{in}}{RT_{in}} \cdot x_a \cdot \frac{V_d N}{60 \cdot x} \cdot Hu \cdot FA \cdot \eta_i \quad \text{and} \quad BP = \frac{\eta_v p_{in}}{RT_{in}} \cdot x_a \cdot \frac{V_d N}{120} \cdot Hu \cdot FA \cdot \eta_i - FP$$

Where η_v and η_i are volumetric and indicated thermal efficiencies

Mechanical Efficiency $\eta_m = \frac{BP}{IP}$

Volumetric Efficiency $\eta_v \equiv \frac{\dot{m}_a}{\dot{m}_{a,ideal}} = \frac{\dot{m}_a}{\frac{\rho V_d N}{60 \cdot x}}$

Mean Effective Pressure (also imep, bmep, fmep)

$$mep \equiv \frac{P}{V_d}, \quad mep = \frac{60 \cdot x \cdot P}{V_d \cdot N} \quad (x=1 \text{ for two stroke, } x=2 \text{ for four stroke})$$

Fuel Air Ratio and Fuel Consumption $FA \equiv \frac{\dot{m}_f}{\dot{m}_a} = \frac{m_f}{m_a}$

$$\dot{m}_f = \frac{P}{Hu \cdot \eta_o}, \quad \text{and} \quad sfc = \frac{\dot{m}_f}{P} = \frac{1}{Hu \cdot \eta_o} \quad sfc \left(\text{in } \frac{g}{kW \cdot hr} \right) = \frac{3.6 \times 10^6}{Hu \cdot \eta_o} \quad (Hu \text{ is in kJ/kg})$$

$$AF = \frac{m_a}{m_f} = \frac{M_a}{M_f} \cdot \frac{p_a}{p_f} = \frac{M_a}{M_f} \cdot \frac{p_{tot} - p_f}{p_f} \quad \text{where } p_f = \text{partial pressure (vapor pressure) of the fuel}$$

Air displacement correction factor $x_a = \frac{1}{1 + FA \cdot \frac{M_a}{M_f} + h \cdot \frac{M_a}{M_w} + \frac{m_{EGR}}{m_a} \cdot \frac{M_a}{M_{EGR}}}$ where $h = \text{specific humidity}$

Brake specific emissions of pollutant 'X' $bsX = \frac{\dot{m}_x (3.6 \times 10^6)}{BP}$

$$\text{where } \dot{m}_x = \dot{m}_{exh} \cdot y_x \cdot \frac{MW_x}{MW_{exh}} \quad \text{and} \quad BP = \frac{2\pi TN}{60000}$$

(over→)

Turbochargers

$$\text{Compressors} \quad W_{c,s=c} = c_p T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad W_{c,actual} = c_p (T_2 - T_1) \quad \eta_c = \frac{W_{c,s=c}}{W_{c,actual}}$$

$$\text{Turbines} \quad W_{t,s=c} = c_p' T_5 \left[1 - \left(\frac{p_6}{p_5} \right)^{\frac{\gamma'-1}{\gamma'}} \right] \quad W_{t,actual} = c_p' (T_5 - T_6) \quad \eta_t = \frac{W_{t,actual}}{W_{t,s=c}}$$

$$\text{Pressure Ratio} \quad \frac{p_2}{p_1} = \left\{ 1 + \eta_t \cdot \eta_c \cdot \eta_m \cdot \frac{T_5}{T_1} \cdot \frac{c_p'}{c_p} \left[1 - \left(\frac{p_6}{p_5} \right)^{\frac{\gamma'-1}{\gamma'}} \right] \right\}^{\frac{\gamma}{\gamma-1}} \quad \text{where } \eta_m \approx 1 \text{ (mechanical efficiency)}$$

Friction

$$\text{Force required to move two parallel plates (fundamental)} \quad F = \tau \cdot A = A \cdot \mu \frac{du}{dy} = A \cdot \frac{U}{\delta y}$$

$$\text{Force required to rotate a bearing (e.g. journal bearings)} \quad F_f = \mu \frac{U \cdot l \cdot D}{C_r} \cdot \frac{\pi}{\sqrt{1-\varepsilon^2}}$$

$$\text{Power required to rotate a bearing} \quad P_f = F_f \cdot U = \mu \frac{U^2 \cdot l \cdot D}{C_r} \cdot \frac{\pi}{\sqrt{1-\varepsilon^2}}$$

$$\text{Force required to move two plane surfaces (e.g. piston rings)} \quad F_l = \frac{6 \cdot \mu \cdot U \cdot a^2}{h_0^2} \left[\ln(1+m) - \frac{2m}{m+2} \right]$$

$$\text{Friction force per unit length} \quad F_f = f \cdot F_l$$

$$\text{Friction coefficient} \quad f = K_l \sqrt{\frac{\mu \cdot U}{F_l}}$$

$$\text{For SI engines} \quad fmep = 97 + 15 \cdot \left(\frac{N}{1000} \right) + 5 \cdot \left(\frac{N}{1000} \right)^2$$

$$\text{For DI (direct injection) engines} \quad fmep = 75 + 48 \cdot \left(\frac{N}{1000} \right) + 0.4 \cdot S_p^2$$

$$\text{For IDI (indirect injection) engines} \quad fmep = 144 + 48 \cdot \left(\frac{N}{1000} \right) + 0.4 \cdot S_p^2$$